Schwartz 11.2(a) $u_1(p)\bar{u}_1(p) + u_2(p)\bar{u}_2(p)$

$$= \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi_1 \\ \sqrt{p\cdot\bar{\sigma}}\,\xi_1 \end{pmatrix} \left(\xi_1^\dagger \sqrt{p\cdot\sigma}^\dagger \quad \xi_1^\dagger \sqrt{p\cdot\bar{\sigma}}^\dagger \right) \gamma_0$$

$$+ \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi_2 \\ \sqrt{p\cdot\bar{\sigma}}\,\xi_2 \end{pmatrix} \left(\xi_2^\dagger \sqrt{p\cdot\sigma}^\dagger \quad \xi_2^\dagger \sqrt{p\cdot\bar{\sigma}}^\dagger \right) \gamma_0$$

$$= \begin{pmatrix} \sqrt{p\cdot\sigma} \\ \sqrt{p\cdot\bar{\sigma}} \end{pmatrix} (\xi_1\xi_1^\dagger + \xi_2\xi_2^\dagger) \left(\sqrt{p\cdot\sigma}^\dagger \quad \sqrt{p\cdot\bar{\sigma}}^\dagger \right) \gamma_0$$

Recall $\xi_1\xi_1^\dagger = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}$, $\xi_2\xi_2^\dagger = \begin{pmatrix} 0 & \\ & 1 \end{pmatrix}$, $\xi_1\xi_1^\dagger + \xi_2\xi_2^\dagger = \mathbb{1}$.

$$= \begin{pmatrix} \sqrt{p\cdot\sigma} \\ \sqrt{p\cdot\bar{\sigma}} \end{pmatrix} \left(\sqrt{p\cdot\sigma}^\dagger \quad \sqrt{p\cdot\bar{\sigma}}^\dagger \right) \gamma_0 \mathbb{1}$$

$$= \begin{pmatrix} \sqrt{p\cdot\sigma}\sqrt{p\cdot\sigma}^\dagger & \sqrt{p\cdot\sigma}\sqrt{p\cdot\bar{\sigma}}^\dagger \\ \sqrt{p\cdot\bar{\sigma}}\sqrt{p\cdot\sigma}^\dagger & \sqrt{p\cdot\bar{\sigma}}\sqrt{p\cdot\bar{\sigma}}^\dagger \end{pmatrix} \gamma_0 \mathbb{1}$$

$$= \begin{bmatrix} \sqrt{(p\cdot\sigma)(p\cdot\bar{\sigma})} & p\cdot\sigma \\ p\cdot\bar{\sigma} & \sqrt{(p\cdot\bar{\sigma})(p\cdot\sigma)} \end{bmatrix} \cancel{\gamma_0} \mathbb{1}$$

$$= \begin{bmatrix} \sqrt{E^2 - (p\cdot\sigma)(p\cdot\sigma)} & p\cdot\sigma \\ p\cdot\bar{\sigma} & \sqrt{E^2 - (p\cdot\sigma)(p\cdot\sigma)} \end{bmatrix} \mathbb{1}$$

$$= p\cdot\gamma + m = \boxed{\not{p} + m}$$

$$v_1(p)\bar{v}_1(p) + v_2(p)\bar{v}_2(p)$$

$$= \begin{pmatrix} \sqrt{p\cdot\sigma}\,\eta_1 \\ -\sqrt{p\cdot\bar{\sigma}}\,\eta_1 \end{pmatrix} (\eta_1^\dagger \sqrt{p\cdot\sigma}^\dagger \quad -\eta_1^\dagger \sqrt{p\cdot\bar{\sigma}}^\dagger)\,\gamma_0$$

$$+ \begin{pmatrix} \sqrt{p\cdot\sigma}\,\eta_2 \\ -\sqrt{p\cdot\bar{\sigma}}\,\eta_2 \end{pmatrix} (\eta_2^\dagger \sqrt{p\cdot\sigma}^\dagger \quad -\eta_2^\dagger \sqrt{p\cdot\bar{\sigma}}^\dagger)\,\gamma_0$$

$$= \begin{pmatrix} \sqrt{p\cdot\sigma} \\ -\sqrt{p\cdot\bar{\sigma}} \end{pmatrix} \times (\eta_1\eta_1^\dagger + \eta_2\eta_2^\dagger) \times (\sqrt{p\cdot\sigma}^\dagger \quad -\sqrt{p\cdot\bar{\sigma}}^\dagger)\,\gamma_0$$

By convention, $\eta_1\eta_1^\dagger + \eta_2\eta_2^\dagger = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} = \mathbb{1}$

$$= \begin{pmatrix} \sqrt{p\cdot\sigma} \\ -\sqrt{p\cdot\bar{\sigma}} \end{pmatrix} (\sqrt{p\cdot\sigma}^\dagger \quad -\sqrt{p\cdot\bar{\sigma}}^\dagger)\,\gamma_0\,\mathbb{1}$$

$$= \begin{pmatrix} p\cdot\sigma & -\sqrt{(p\cdot\sigma)(p\cdot\bar{\sigma})} \\ -\sqrt{(p\cdot\sigma)(p\cdot\bar{\sigma})} & p\cdot\bar{\sigma} \end{pmatrix} \gamma_0\,\mathbb{1}$$

$$= \begin{pmatrix} -m^2 & p\cdot\sigma \\ p\cdot\bar{\sigma} & -m^2 \end{pmatrix} \mathbb{1}$$

$$= \gamma p - m^2 = \boxed{\not{p} - m}$$

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Schwartz 11-2(b) $\bar{u}_\sigma(p)\gamma^\mu u_{\sigma'}(p) = ?$

Consider $\sum_{\sigma'}[\bar{u}_\sigma(p)\gamma^\mu u_{\sigma'}(p)]\bar{u}_{\sigma'}(p)$

$$= \cancel{(\not{p}+m)}\ \bar{u}_\sigma(p)\gamma^\mu \sum_{\sigma'}[u_{\sigma'}(p)\bar{u}_{\sigma'}(p)]$$

$$= \bar{u}_\sigma(p)\gamma^\mu[\not{p}+m]$$

$$= \bar{u}_\sigma(p)\gamma^\mu[\gamma^\alpha p_\alpha + m]$$

$$= \bar{u}_\sigma(p)\left[2g^{\mu\alpha}p_\alpha - \gamma^\alpha\gamma^\mu p_\alpha + \gamma^\mu m\right]$$

$$= \bar{u}_\sigma(p)\left[2p^\mu - \gamma^\mu[\not{p}-m]\right]$$

$0 = \not{p} - m$ is the equation satisfied by u, thus we have

$$= \bar{u}_\sigma(p)(2p^\mu)$$

$$\Rightarrow \sum_{\sigma'}[\bar{u}_\sigma(p)\gamma^\mu u_{\sigma'}(p)]\bar{u}_{\sigma'}(p) = 2p^\mu\bar{u}_\sigma(p)$$

$$\Rightarrow \boxed{\bar{u}_\sigma(p)\gamma^\mu u_{\sigma'}(p) = 2\delta_{\sigma'\sigma}p^\mu}$$

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